PAG. 165

Dimostrazione Equazione IPERBOLE

$|c-x|$ & $|x - -c|$

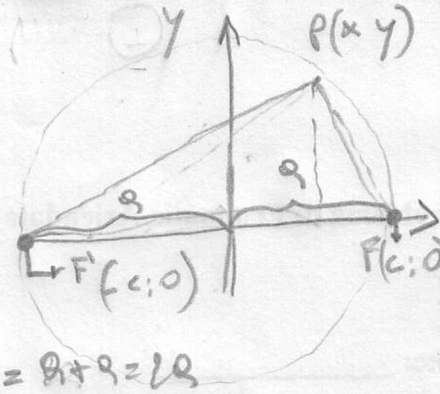
$\downarrow$

$-(c-x)$ $\quad (x+c)$

$\downarrow$

$(x-c)$

$\downarrow$



$PF' - PF' = a+a = 2a$

Dunque: $(c-x)^2 = (x-c)^2 \rightarrow \cancel{c^2} - \cancel{2cx} + \cancel{x^2} = \cancel{x^2} - \cancel{2cx} + \cancel{c^2}$

Per cui:

$PF' = \sqrt{(x+c)^2+y^2}$ & $\begin{cases} PF = \sqrt{(x-c)^2+y^2} \\ PF = \sqrt{(c-x)^2+y^2} \end{cases}$

$\sqrt{(c-x)^2+y^2} - \sqrt{(x+c)^2+y^2} = 2a$

$\left(\sqrt{(c-x)^2+y^2}\right)^2 = \left(2a + \sqrt{(x+c)^2+y^2}\right)^2$

$(c-x)^2+y^2 = 4a^2 + 4a\sqrt{(x+c)^2+y^2} + (x+c)^2+y^2$

$\cancel{c^2} - 2cx + \cancel{x^2} + \cancel{y^2} = 4a^2 + 4a\sqrt{(x+c)^2+y^2} + \cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2}$

$-2cx - 2cx - 4a^2 = -4a\sqrt{x^2+2cx+c^2+y^2}$

$\left(+\cancel{4}cx + \cancel{4}a^2\right)^2 = \left(-\cancel{4}a\sqrt{x^2+2cx+c^2+y^2}\right)^2$

$c^2x^2 + \cancel{2a^2cx} + a^4 = a^2x^2 + \cancel{2a^2cx} + a^2c^2 + a^2y^2$

$-a^2y^2 + c^2x^2 - a^2x^2 - a^2c^2 + a^4 = 0$

$x^2(c^2-a^2) - a^2y^2 - a^2(c^2-a^2) = 0$

$x^2(c^2-a^2) - a^2y^2 = a^2(c^2-a^2)$

$b^2 = c^2 - a^2$

$b^2x^2 - a^2y^2 = a^2b^2$

Dividendo x $a^2b^2 \Rightarrow$

$$\frac{\cancel{b^2}x^2}{a^2\cancel{b^2}} - \frac{\cancel{a^2}y^2}{\cancel{a^2}b^2} = \frac{\cancel{a^2b^2}}{\cancel{a^2b^2}}$$

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

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